Some Recent Results on Technicolor and Extended Technicolor Models

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Outline

- Basics of technicolor (TC) and extended technicolor (ETC)

- UV to IR evolution and walking TC

- Calculation of higher-loop corrections to UV $\longrightarrow$ IR evolution, α_{IR}, γ for technifermions in various representations, comparison with lattice results

- Some phenomenological constraints on TC/ETC models, including flavor (collider, precision electroweak, and dark matter constraints largely covered in other talks)

- TC/ETC models with color-singlet technifermions

- Conclusions

Basics of Technicolor

Technicolor (TC) is an asymptotically free vectorial gauge theory with gauge group G_{TC}, which we will take to be $\mathrm{SU}(N_{TC})$, and a set of fermions $\{F\}$ transforming according to some representation(s) of G_{TC}. The TC interaction becomes strong at a scale Λ_{TC} of order the electroweak scale, confining and producing technifermion condensates, yielding dynamical electroweak symmetry breaking (EWSB) (Weinberg, Susskind, 1979).

Motivations:

- In contrast to the Standard Model (SM), where the EWSB is put in by hand, via $\mu^2 < 0$ in the Higgs potential $V = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$, in TC the EWSB is an automatic result of the technifermion condensate formation.

- In two previous cases where fundamental scalar fields were used to model spontaneous symmetry breaking, the actual underlying physics did not involve fundamental scalar fields but instead bilinear fermion condensates:

(i) Superconductivity: Ginzburg-Landau free energy functional used complex scalar field ϕ, but the actual origin of SC is the dynamical formation of a condensate of Cooper pairs.

(ii) The σ model for spontaneous chiral symmetry breaking (SχSB) in hadronic physics attributed this to the vev of a scalar field, σ, but the actual origin of SχSB in QCD is the dynamical formation of a $\langle \bar{q} q \rangle$ condensate.

Indeed, this $\langle \bar{q} q \rangle$ condensate in QCD breaks EW symmetry, although the scale, f_π, is 10^{-3} of the EWSB scale of ~ 250 GeV. (A gedanken world in which this EWSB by QCD were the only source of EWSB would have many exotic properties; see, e.g., Quigg and RS, Phys. Rev. D 79, 096002 (2009)).

Another motivation: since there are no fundamental scalar fields in TC, there is no hierarchy problem.

For TC model-building, we arrange that left-handed technifermions form one or more SU(2)$_L$ doublets, with some weak hypercharge, and with corresponding right-handed components. A minimal choice is to restrict to one SU(2)$_L$ doublet,

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}_L \qquad F_{1R}, \quad F_{2R}$$

where F_1, F_2 transform (vectorially) according to some rep. R of the TC gauge group. In general, a model may also contain technifermions that are SM singlets (e.g., to get walking behavior). Typically, the Lagrangian masses of technifermions are taken to be zero, although they may be nonzero if they are EW-singlets.

The SU(N_{TC}) TC theory is asymptotically free, so as energy scale decreases, α_{TC} increases, eventually producing condensates; for generic N_{TC}, these are $\langle \bar{F}_1 F_1 \rangle$, $\langle \bar{F}_2 F_2 \rangle$ transforming as $I_w = 1/2$, $|Y| = 1$, breaking EW symmetry at Λ_{TC}.

Just as in the QCD example above, the W and Z pick up masses, but now involving the TC scale:

$$m_W^2 \simeq \frac{g^2\, F_{TC}^2\, N_D}{4}\,, \qquad m_Z^2 \simeq \frac{(g^2 + g'^2)\, F_{TC}^2\, N_D}{4}$$

satisfying the tree-level relation $\rho = 1$, where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$, because of the I_w and Y of $\langle \bar{F} F \rangle$. Here $F_{TC} \sim \Lambda_{TC}$ is the TC analogue to $f_\pi \sim \Lambda_{QCD}$ and $N_D =$ number of SU(2)$_L$ technidoublets. For minimal model, $N_D = 1$, so $F_{TC} = 250$ GeV.

Another class of TC models that has been studied in the past uses one SM family of technifermions (1FTC) (in fund. rep. of G_{TC})

$$\begin{pmatrix} U^{a\tau} \\ D^{a\tau} \end{pmatrix}_L \quad U_R^{a\tau}, \quad D_R^{a\tau}$$

$$\begin{pmatrix} N^{\tau} \\ E^{\tau} \end{pmatrix}_L \quad N_R^{\tau}, \quad E_R^{\tau}$$

(a, τ color, TC indices) with usual Y assignments. Similar condensate formation, with approx. equal condensates $\langle \bar{F} F \rangle$ for $F = U^a, \ D^a, \ N, \ E$, generating dynamical technifermion masses $\Sigma_{TC} \sim \Lambda_{TC}$, analogous to constituent quark mass $\sim \Lambda_{QCD}$ in QCD. Resultant m_W^2 and m_Z^2 given by formula above with $N_D = N_c + 1 = 4$, so $F_{TC} \simeq 125$ GeV for 1FTC.

Several models have been studied where technifermions transform according to the fundamental representation of G_{TC}; also interest in models where technifermions transform as higher reps., e.g., $G_{TC} = \mathrm{SU}(2)$ with $N_f = 2$ adjoint reps. which can produce walking (Sannino talk).

TC is appealing, but, by itself, is not a complete theory; to give masses to quarks and leptons (which are technisinglets), one must communicate the EWSB in the TC sector to these SM fermions. For this, one embeds TC in a larger, extended technicolor (ETC) gauge theory with ETC gauge bosons transforming SM fermions into technifermions (Dimopoulos and Susskind; Eichten and Lane, 1979-80).

An ETC theory is much more ambitious than the SM or MSSM because a successful ETC model would predict the entries in the SM fermion mass matrices and the resultant values of the quark and lepton masses and mixings. It would explain longstanding mysteries like the mass ratios m_e/m_μ, m_u/m_d, m_d/m_s, etc.

Not surprisingly, no fully realistic ETC model has yet been constructed, and TC/ETC models face many stringent constraints. Some of these cause tension with various TC/ETC models, as we will discuss.

One's assessment of TC/ETC model-building depends on how stringently one defines success; if one requires that the model reproduce fermion masses and mixings in detail, then, one would be pessimistic with existing models. But if one regards these models as having a grain of truth (dynamical EWSB), then one can be more optimistic. Clearly, models must not conflict with flavor and precision EW constraints.

To satisfy constraints on flavor-changing neutral current (FCNC) processes, ETC gauge bosons must have large masses. These masses are envisioned as arising from sequential breaking of the ETC chiral gauge symmetry and typically form a hierarchy of three scales.

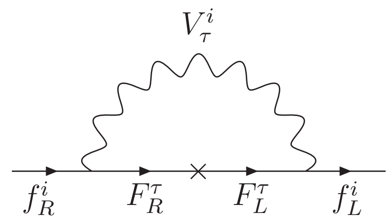
Illustrative ETC breaking scales that have been used:

$$\Lambda_1 \sim 500 - 10^3 \text{ TeV}, \qquad \Lambda_2 \sim 50 - 100 \text{ TeV}, \qquad \Lambda_3 \sim \text{few TeV},$$

The ETC theory is constructed to be asymptotically free, so as the energy decreases from a high scale, ETC coupling α_{ETC} grows, eventually becomes large enough to form condensates that sequentially break the ETC symmetry in three stages, to a residual exact subgroup, which is the TC gauge group; so $G_{ETC} \supset G_{TC}$.

Mass Generation Mechanism for Fermions

The ETC gauge bosons enable SM fermions, which are TC singlets, to transform into technifermions and back. This provides a mechanism for generating SM fermion masses. The figure shows a one-loop graph contributing to diagonal entries in mass matrix for a SM fermion f^i of the i'th generation (suppressing color indices for quarks):



Rough estimate:

$$M_{ii}^{(f)} \simeq \frac{2\alpha_{ETC} C_2(R)}{\pi} \int dk^2 \frac{k^2 \Sigma_{TC}(k^2)}{[k^2 + \Sigma_{TC}(k^2)^2][k^2 + M_i^2]}$$

where $M_i \simeq (g_{ETC}/2)\Lambda_i \simeq \Lambda_i$ is the mass of the ETC gauge bosons that gain mass at scale Λ_i, $C_2(R)$ = quadratic Casimir invariant. With walking technicolor (WTC), $\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)^2/k$ for Euclidean $k \gg \Lambda_{TC}$ in walking regime; contrast with QCD, where $\Sigma(k) \simeq \Sigma(0)^3/k^2$ for $k \gg \Lambda_{QCD}$.

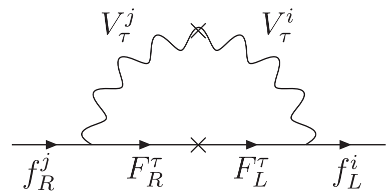
This gives

$$M_{ii}^{(f)} \simeq \frac{\kappa\, C_2(R)\, \eta_i\, \Lambda_{TC}^3}{\Lambda_i^2}$$

where $\kappa \simeq O(10)$ is a numerical factor from the integral and η_i is an RG running factor. This is only a rough estimate, since ETC coupling is strong, so higher-order diagrams are also important.

The sequential breaking of the ETC symmetry at the highest scale Λ_1, the intermediate scale Λ_2, and the lowest scale Λ_3 thus produces the generational hierarchy in the fermion masses. Since these ETC scales enter as inverse powers in the resultant SM fermion masses and since Λ_1 is the largest ETC scale, it follows that first-generation fermion masses are the smallest, and since Λ_3 is the smallest ETC scale, third-generation fermion masses are the largest.

There are mixings among the interaction eigenstates of the ETC gauge bosons to form mass eigenstates. These involve mixings $V_\tau^j \longrightarrow V_\tau^i$, where $i, j \in \{1, 2, 3\}$ and τ are TC indices. Insertions of these on ETC gauge boson lines lead to off-diagonal elements of the $M^{(f)}$ via diagrams like



This yields

$$M_{ij}^{(f)} \simeq \frac{\kappa\, \eta\, \Lambda_{TC}^3\, {}^{j}_{\tau}\Pi^{i}_{\tau}}{\Lambda_i^2 \Lambda_j^2}$$

where ${}^{j}_{\tau}\Pi^{i}_{\tau}$ is the nondiagonal vacuum polarization tensor function representing the ETC gauge boson mixing $V_\tau^i \longrightarrow V_\tau^j$. Diagonalization of the full SM fermion mass matrices yields masses and mixings (e.g., Appelquist, RS, Phys. Lett. B 548, 204 (2002), Appelquist, Piai, RS, Phys. Rev. D 69, 015002 (2004)).

In models such as these in which SM fermion masses arise dynamically, there is a prediction for the asymptotic momentum dependence of the running mass $m_{f_i}(p)$ of a SM fermion of generation i (Christensen, RS, Phys. Rev. Lett. 94, 241801 (2005)): $m_{f_i}(p)$ is constant up to $M_i \simeq \Lambda_i$ and has the power-law decay

$$m_{f_i}(p) \sim m_{f_i(0)} \frac{\Lambda_i^2}{p^2}$$

for Euclidean momenta $p \gg \Lambda_i$. Here we neglect logarithmic factors, which are subdominant relative to this power-law falloff.

Thus, e.g., the third-generation quark masses $m_t(p)$ and $m_b(p)$ decay like Λ_3^2/p^2 for $p \gg \Lambda_3$, while the first-generation quark masses $m_u(p)$ and $m_d(p)$ are hard up to the much higher scale Λ_1, eventually decaying like Λ_1^2/p^2 for $p \gg \Lambda_1$.

UV to IR Evolution and Walking TC

TC models that behaved simply as scaled-up versions of QCD were excluded by their inability to produce sufficiently large fermion masses (especially for the third generation) without having ETC scales so low as to cause excessively large FCNC effects.

As a necessary condition to be viable, modern TC theories are designed to have a coupling $g_{_{TC}}$ that gets large, but runs slowly ("walks") over an extended interval of energy (WTC) (Holdom, Yamawaki et al., Appelquist, Wijewardhana,...).

This behavior arises naturally from an approximate IR zero of the two-loop beta function:

$$\beta(\alpha_{_{TC}}) = \frac{d\alpha_{TC}}{dt} = -\frac{\alpha_{_{TC}}^2}{2\pi}\left(b_1 + \frac{b_2\,\alpha_{_{TC}}}{4\pi} + O(\alpha_{_{TC}}^2)\right)$$

where $t = \ln\mu$, with $b_1 > 0$ - asymp. freedom. For sufficiently many technifermions, $b_2 < 0$, so β has a zero away from the origin at $\alpha_{_{TC}} = -4\pi b_1/b_2 \equiv \alpha_{_{IR}}$.

If $N_f < N_{f,cr}$ (depending on technifermion rep. of G_{TC}, R), as the theory evolves from the UV to IR, α_{TC} gets large, but runs slowly because β approaches this zero at α_{IR}. For TC, we want to choose N_f so that α_{IR} is slightly greater than the minimal value α_{cr} for technifermion condensation. Then the TC theory has quasi-conformal (walking) behavior, with a large $\alpha_{TC}(\mu)$ over an extended interval of energies μ.

As $\alpha_{TC}(\mu)$ eventually exceeds α_{cr} at $\mu \sim \Lambda_{TC}$, the technifermion condensate $\langle \bar{F}F \rangle$ forms, the technifermions gain dynamical masses, and in the low-energy theory at smaller μ, they are integrated out, so the TC beta function changes, and α_{TC} evolves away from α_{IR} which is thus an approximate IR fixed point (IRFP).

For $N_f > N_{f,cr}$, the theory would evolve from the UV to the IR in a chirally symmetric manner, without ever producing $\langle \bar{F}F \rangle$, so the (initially massless) technifermions remain massless, and the IRFP is exact. This conformal regime is of basic field-theoretic interest, although for TC model-building, we should choose the technifermion content so that we are in the phase with SχSB, as is necessary for EWSB.

Walking TC has several desirable features.

- SM fermion masses are enhanced by the factor

$$\eta_i = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu}\,\gamma(\alpha_{TC}(\mu))\right]$$

where the TC theory has walking up to Λ_w; if $\gamma \simeq 1$, this yields $\eta_i \simeq \Lambda_i/\Lambda_{TC}$;

- hence, one can increase ETC scales Λ_i for a fixed m_{f_i}, reducing FCNC effects;
- can reduce electroweak $\hat{S}$ (per doublet) relative to QCD-like value

Pioneering studies to estimate $N_{f,cr}$ used Dyson-Schwinger (DS) equation for the (techni)fermion propagator (Appelquist, Lane, Wijewardhana, Yamawaki..); for $\alpha > \alpha_{cr}$, this yields a nonzero solution for a dynamically generated fermion mass. Simple ladder approx. to DS eq. gives $\alpha_{cr} C_2(R) \sim O(1)$, where R is fermion rep.

As number of technifermions, N_f, increases, $\alpha_{_{IR}}$ decreases, and $N_f \nearrow N_{f,cr}$ as $\alpha_{_{IR}} \searrow \alpha_{cr}$. This yielded the estimate $N_{f,cr} \simeq 4N_{TC}$.

Lattice gauge simulations provide a fully nonperturbative determination of $N_{f,cr}$ and measurement of the anomalous dimension γ that describes the running of m and the bilinear operator, $\bar{F}F$ as a function of $\ln\mu$. In recent years, intensive work using lattice methods to determine these quantities for SU(3), SU(2), and various fermion representations, including fundamental, adjoint, and 2-index symmetric tensor rep. with latest results reported at this conference.

Higher-loop corrections to UV $\rightarrow$ IR evolution of a TC gauge theory

Because of the strong-coupling nature of the physics at an approximate IRFP of interest to TC theories, there are generically significant higher-order corrections to results obtained from the two-loop β function.

This motivates the calculation of the location of the IR zero in β and the value of $\gamma = \gamma(\alpha)$ evaluated at $\alpha = \alpha_{IR}$ to higher-loop order. We have done this to 3-loop and 4-loop order (Ryttov and RS, PRD 83, 056011 (2011), arXiv:1011.4542; see also Pica and Sannino, PRD 83,035013 (2011), arXiv:1011.5917).

Although the coefficients in the beta function at 3-loop and higher-loop order are scheme-dependent, the results give a measure of the accuracy of the 2-loop calculation of the IR zero, and similarly with the value of γ evaluated at this IR zero. We use the $\overline{MS}$ scheme, for which the coefficients of β and γ have been calculated up to 4-loop order by Vermaseren, Larin, and van Ritbergen.

The greater accuracy obtained with 3-loop and 4-loop calculations in QCD has been amply demonstrated by the excellent fit to data that has been achieved for $\alpha_s(\mu)$ in QCD (e.g., Bethke, Eur. Phys. J. C64, 689 (2009)).

Analytic and numerical results are presented in our paper; here we only list numerical results. We find that for given SU(N) ($N \equiv N_{TC}$) and fermion content for which $\exists$ IR zero of β, the 3- and 4-loop values of α_{IR} are somewhat smaller than the 2-loop value.

Results for N_f technifermions in the fundamental rep. of SU(N) for $N = 2,\ 3$:

N	N_f	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$	$\alpha_{IR,4\ell}$
2	7	2.83	1.05	1.21
2	8	1.26	0.688	0.760
2	9	0.595	0.418	0.444
2	10	0.231	0.196	0.200
3	10	2.21	0.764	0.815
3	11	1.23	0.578	0.626
3	12	0.754	0.435	0.470
3	13	0.468	0.317	0.337
3	14	0.278	0.215	0.224
3	15	0.143	0.123	0.126
3	16	0.0416	0.0397	0.0398

Similarly, we find that for given N, R, and N_f, the value of γ calculated to 3-loop and 4-loop order and evaluated at the value of α_{IR} calculated to the same order is somewhat smaller than the 2-loop value:

For N_f technifermions in $R =$ fundamental rep. of SU(N) for $N = 2,\ 3$:

N	N_f	$\gamma_{2\ell}(\alpha_{IR,2\ell})$	$\gamma_{3\ell}(\alpha_{IR,3\ell})$	$\gamma_{4\ell}(\alpha_{IR,4\ell})$
2	7	(2.67)	0.457	0.0325
2	8	0.752	0.272	0.204
2	9	0.275	0.161	0.157
2	10	0.0910	0.0738	0.0748
3	10	(4.19)	0.647	0.156
3	11	1.61	0.439	0.250
3	12	0.773	0.312	0.253
3	13	0.404	0.220	0.210
3	14	0.212	0.146	0.147
3	15	0.0997	0.0826	0.0836
3	16	0.0272	0.0258	0.0259

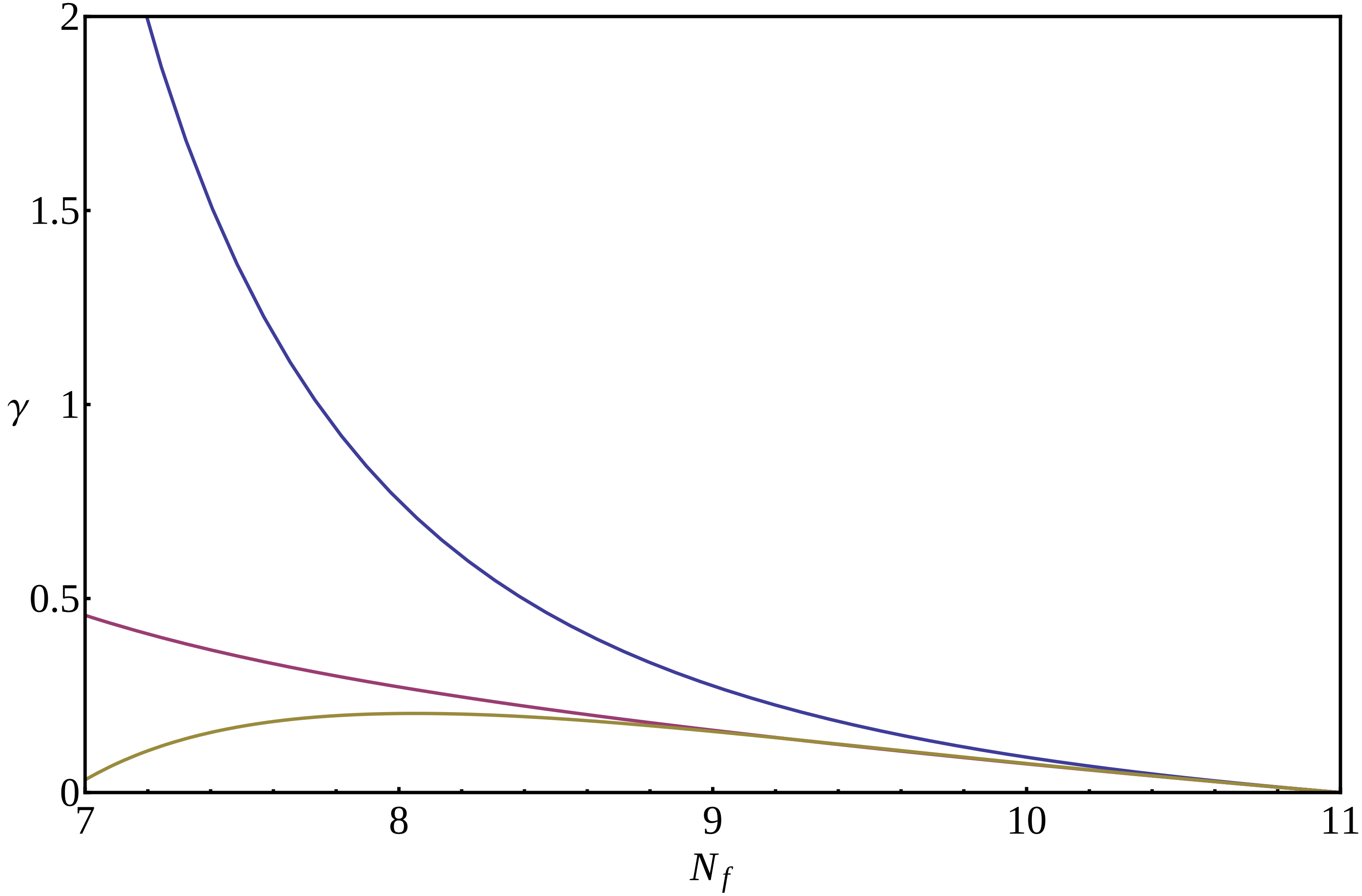


Figure 1: Anomalous dimension γ for SU(2) for N_f fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ($N_{f,max} = 11$).

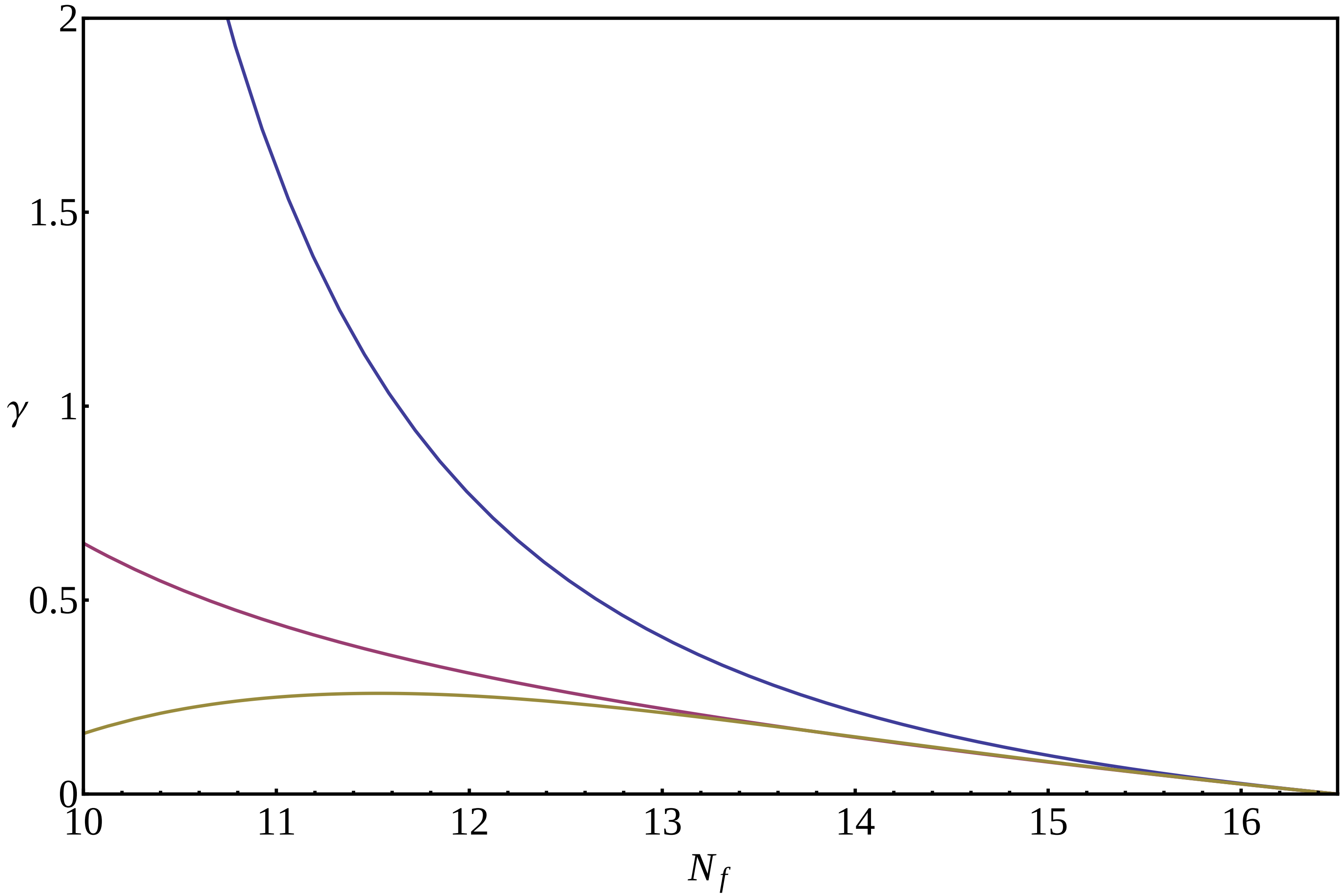


Figure 2: Anomalous dimension $\boldsymbol{\gamma}$ for SU(3) for $\boldsymbol{N_f}$ fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ($\boldsymbol{N_{f,max} = 16.5}$).

Some caveats: (i) as N_f decreases toward $N_{f,cr}$, α_{IR} gets large, so perturbative calcs. of α_{IR} and γ are less reliable. Values of γ in parentheses are unphysically large.

(ii) In the phase with confinement and SχSB, α_{IR} is only an approximate IRFP and γ is only an effective quantity describing the theory at scales μ where α is near to α_{IR}. In the conformal phase, an IRFP is exact (although our perturbative calculation of it is only approximate), and γ describes the scaling of the bilinear $\bar{F}F$ at this IRFP in a scheme-independent manner. For $N_f \simeq N_{f,cr}$ it is difficult to determine whether a given theory is in the chirally broken or symmetric phase, e.g., with lattice methods.

Some examples of comparison with lattice measurements:

For SU(3) with $N_f = 12$, from the table above,

$$\gamma_{IR,2\ell} = 0.77, \qquad \gamma_{IR,3\ell} = 0.31, \qquad \gamma_{IR,4\ell} = 0.25$$

Lattice results

$\gamma = 0.414 \pm 0.016$ (Appelquist, Fleming, Lin, Neil, Schaich, PRD 84, 054501 (2011), arXiv:1106.2148, analyzing data of Kuti et al., PLB 703, 348 (2011), arXiv:1104.3124, inferring consistency with conformality)

$\gamma \sim 0.35$ (DeGrand, arXiv:1109.1237, also analyzing Kuti et al. data).

So here the 2-loop value is slightly larger than, and the 3-loop and 4-loop values closer to, these lattice measurements.

For SU(3) with $N_f = 10$ fermions in the fundamental rep., the perturbative calculations up to 4-loops are less reliable, because of the larger value of α_{IR}. This is clear from the large change in γ as one goes from $\ell = 2$ to $\ell = 4$ loops; for the two highest loop orders, from table above:

$$\gamma_{IR,3\ell} = 0.65, \qquad \gamma_{IR,4\ell} = 0.16$$

Lattice results:

$0.50 < \gamma < 1.0$ at a IRFP (Hayakawa et al.; Yamada, priv. commun. and talk at this conf.)

Forthcoming measurements for SU(3) with $N_f = 10$ from other groups, including Appelquist, Fleming, Neil.. (LSD Collab.); . Aoyama, Ikeda, Itou, Kurachi, et al...

Clearly, lattice methods are the most powerful for determining $N_{f,cr}$ for various SU(N) theories and (i) the effective behavior of γ in the vicinity of an approximate IRFP in the chirally broken phase, of interest for walking TC theories, and (ii) the value of γ at an exact IRFP in the conformal phase.

We have also carried out these higher-loop calculations for fermions in larger representations. For the adjoint representation:

N	$\alpha_{IR,2\ell,adj}$	$\alpha_{IR,3\ell,adj}$	$\alpha_{IR,4\ell,adj}$
2	0.628	0.459	0.493
3	0.419	0.306	0.323

N	$\gamma_{2\ell,adj}(\alpha_{IR,2\ell,adj})$	$\gamma_{3\ell,adj}(\alpha_{IR,3\ell,adj})$	$\gamma_{4\ell,adj}(\alpha_{IR,4\ell,adj})$
2	0.820	0.543	0.571
3	0.820	0.543	0.561

For SU(2) with $N_f = 2$ fermions in the adjoint rep., lattice results include (caution: various groups quote uncertainties differently):

$\gamma = 0.22 \pm 0.06$ (Del Debbio et al., PRD 82, 014510 (2010))

$\gamma = 0.49 \pm 0.13$ (Catterall, Del Debbio et al., arXiv:1010.5909, PoS(Lat2010) 057)

$\gamma = 0.31 \pm 0.06$ (DeGrand, Shamir, Svetitsky, PRD 83, 074507 (2011)

$\gamma = 0.17 \pm 0.05$ (Appelquist et al., PRD 84, 054501 (2011), arXiv:1106.2148)

$-0.6 < \gamma < 0.6$ (Catterall, Del Debbio, et al., arXiv:1108.3794)

Further, we have done calculations for fermions in the symmetric and antisymmetric rank-2 tensor representations of SU(N), and there are studies of SU(3) with $N_f = 2$ fermions in the symmetric rank-2 tensor representation (sextet rep.) by DeGrand, Shamir, Svetitsky; Kogut and Sinclair.

Lattice results on $N_{f,cr}$ and quasi-conformal behavior by other groups (Deuzeman, Lombardo, Pallante, Hasenfratz, Hietanen et al...); here we have only cited recent measurements of γ.

Some Constraints on TC/ETC Models

Early studies of ETC by many authors, and some recent papers, have considered the TC theory as an effective low-energy theory and added various plausible four-fermion operators linking SM fermions and technifermions.

Part of our work has focused on constructing reasonably ultraviolet-complete ETC models that predict the forms and coefficients of the four-fermion operators in the effective low-energy technicolor theory. One can gain insight from these models, bearing in mind, however, that they are not fully realistic.

ETC theories naturally gauge the generational indices of SM fermions and combine them with TC indices. Some ETC gauge bosons thus transform SM fermions into technifermions and vice versa. Commutators of generators of the Lie algebra for the ETC group then also lead to ETC gauge bosons that change generations, via processes such as $f^i_L \to f^j_L + V^i_j$ and $f^i_R \to f^j_R + V^i_j$, where f^i is a SM fermion of i'th generation, and V^i_j is an ETC gauge boson.

Typically, ETC is arranged to be an asymptotically free chiral gauge theory, and includes a set of SM-singlet, ETC-nonsinglet fermions chosen so that as the scale decreases from the deep UV, the ETC gauge coupling becomes large enough to produce condensates of these SM-singlet fermions, which break the ETC gauge symmetry.

Since this involves strongly coupled gauge interactions, it is not precisely calculable, but the pattern of condensate formation can be plausibly determined by the most attractive channel (MAC) criterion. Some studies include Appelquist and Terning, PRD 50, 2116 (1994); Appelquist and Evans, PRD 53, 2789 (1996); Appelquist, Evans, Selipsky, PLB 374, 145 (1996); Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003); Appelquist, Piai, RS, PRD 69, 015002 (2004); Christensen and RS, PRD 74, 015004 (2006); Ryttov and RS PRD 81, 115013 (2010); PRD 84, 056009 (2011).

To account for the three generations of SM fermion masses, there is a sequential breaking of the ETC gauge symmetry, at the three scales Λ_i, $i = 1,\ 2,\ 3$. Although the full ETC theory is chiral, we focus here on ETC models with vectorial couplings to quarks and charged leptons, denoted VSM ETC models.

At the highest scale, Λ_1, G_{ETC} breaks to H_{ETC}, and the gauge bosons in the coset G_{ETC}/H_{ETC} gain masses $\sim g_{_{ETC}}\Lambda_1 \sim \Lambda_1$, and so forth for the breakings at the two lower scales Λ_2 and Λ_3.

Studies of reasonably UV-complete models showed how not just diagonal, but also off-diagonal, elements of SM fermion mass matrices could be produced, via nondiagonal propagator corrections to ETC gauge bosons, $V^i_\tau \to V^j_\tau$, where $i,\ j$ are generation indices and τ is a TC index (Appelquist, Piai, RS, PRD 69, 015002 (2004)).

A feature that was found in these studies of reasonably UV-complete ETC models was the presence of approximate residual generational symmetries that naturally suppress these ETC gauge boson propagator corrections and hence also off-diagonal elements of SM fermion mass matrices.

Further, a possible mechanism to account for the very small neutrino masses was presented. This made use of suppressed Dirac and Majorana neutrino masses leading to a low-scale seesaw (Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003)).

TC/ETC theories are constrained by FCNC processes. These can be suppressed by making the ETC breaking scales Λ_i sufficiently large, but this is restricted by the requirement that one not cause excessive suppression of SM fermion masses.

One insight from studies of reasonably UV-complete ETC models was that the approximate residual generational symmetries suppress the FCNC effects.

For example, consider $K^0 - \bar{K}^0$ mixing and resultant $K_L - K_S$ mass difference $\Delta m_{K_L K_S}$. SM contribution consistent with experimental value $\Delta m_{K_L K_S}/m_K \simeq 0.7 \times 10^{-14}$.

Simple effective Lagrangian used in early studies without a UV-complete ETC theory: $\mathcal{L}_{eff} = c[s\gamma_\mu d]^2$ with coefficient $c \sim 1/\Lambda_{ETC}^2$, usually with just a single generic ETC scale.

Now in terms of ETC eigenstates, an $s\bar{d}$ in a $\bar{K}^0$ produces a V_1^2 ETC gauge boson, but this cannot directly yield a $d\bar{s}$ in the final-state K^0; the latter is produced by a V_2^1. So this requires either the ETC gauge boson mixing $V_1^2 \to V_2^1$ or the related mixing of ETC quark eigenstates to produce mass eigenstates.

The ETC gauge boson propagator insertion ${}_2^1\Pi_1^2$ required for this breaks the generational symmetries associated with the $i = 1$ and $i = 2$ generations, and hence

$$|{}_2^1\Pi_1^2| \lesssim \Lambda_2^2$$

Therefore, the contribution to $\bar{K}^0 \to K^0$ transition from $V_1^2 \to V_2^1$:

$$|c| \lesssim \frac{1}{\Lambda_1^2}\,{}_2^1\Pi_1^2\,\frac{1}{\Lambda_1^2} \sim \frac{\Lambda_2^2}{\Lambda_1^2}\,\frac{1}{\Lambda_1^2} \ll \frac{1}{\Lambda_1^2}$$

With above values for Λ_1 and Λ_2, the suppression factor is $(\Lambda_2/\Lambda_1)^2 \simeq 10^{-2}$. So rather than the naive result $\Delta m_{K_L K_S}/m_K \sim \Lambda_{QCD}^2/\Lambda_1^2$, this yields the considerably smaller result

$$\frac{\Delta m_{K_L K_S}}{m_K} \sim \frac{\Lambda_2^2\,\Lambda_{QCD}^2}{\Lambda_1^4} \sim 10^{-15}$$

which agrees with experimental limits on new-physics contributions.

Similar analysis applies to ETC contributions to a number of other FCNC processes ($D^0-\bar{D}^0$, $B_d^0-\bar{B}_d^0$, $B_s^0-\bar{B}_s^0$ mixing, $b \longrightarrow s\gamma$, $\mu \longrightarrow e\gamma$...), etc. Some studies of FCNC constraints that take account of these approximate generational symmetries include Appelquist, Piai, RS, PLB 593, 175 (2004); PLB 595, 442 (2004); Appelquist, Christensen, Piai, RS, PRD 70, 093010 (2004).

It remains challenging to construct a TC/ETC model (e.g. VSM type) that does everything that is demanded of it, including sufficient suppression of FCNC effects and accounting for realistic quark, charged lepton, and neutrino masses and quark and lepton mixing.

One particular aspect of this concerns achieving not just inter-generational, but also intra-generational, mass splittings. One must explain why the quarks are heavier than the charged leptons in each generation. One must also explain, why $m_c >> m_s$ and $m_t >> m_b$. The large $m_t >> m_b$ mass splitting is especially difficult to achieve without excessive contributions to ρ (equiv. T).

Examples of attempts: one might try to achieve this splitting using a class of ETC models in which left and right components of up-type quarks and techniquarks transform the same way under SU(5)$_{ETC}$ but the left and right components of down-type quarks and techniquarks transform according to relatively conjugate reps. However, these are excluded because of excessive FCNC's (Appelquist, Piai, RS, op. cit., Appelquist, Christensen, and Piai, RS, op. cit.).

One might also try to use two ETC groups, arranged so that the c and t get their masses direction via one ETC exchange, but the masses of the s and b (as well as of the charged leptons) require mixing between the two ETC groups and hence are suppressed (Appelquist, Evans, Selipsky, op. cit., Christensen, RS, op cit.). This can work for $t - b$ mass splitting, but encounters difficulties when trying to fit all of the quark and lepton masses.

Another possible approach is to mix the b with an SU(2)$_L$-singlet vectorlike b', which would require further ingredients in a UV completion.

The difficulty of explaining $t - b$ splitting while satisfying other constraints motivated the development of a different kind of EWSB model than conventional TC/ETC theories, namely topcolor-assisted technicolor (TC2) theories (Hill, Bardeen, Chivukula, Simmons, Eichten, Lane, Martin,..; antecedents go back to Nambu Jona-Lasinio four-fermion interactions for SχSB.

TC2 models use separate asymptotically free, vectorial SU(3) gauge interactions acting on the third generation of quarks and on the first two generations of quarks, denoted as SU(3)$_1$ and SU(3)$_2$, respectively. The SU(3)$_1$ interaction becomes sufficiently strong, at a scale Λ_t of order 1 TeV, and produces a $\langle \bar{t}t \rangle$ condensate. This condensate is primarily responsible for the top mass.

The SU(3)$_1$ interaction actually treats the t and b quarks in the same way and hence, by itself, would also produce a $\langle \bar{b}b \rangle \simeq \langle \bar{t}t \rangle$, and hence $m_b \simeq m_t$. This is prevented by an additional set of hypercharge-type $\mathrm{U}(1)_1 \otimes \mathrm{U}(1)_2$ gauge interactions; the $\mathrm{U}(1)_1$ is strong and attractive (repulsive) in the $\bar{t}t$ ($\bar{b}b$) channels.

The $\mathrm{SU}(3)_1 \otimes \mathrm{SU}(3)_2$ and $\mathrm{U}(1)_1 \otimes \mathrm{U}(1)_2$ symmetries are each assumed to break to their respective diagonal subgroups, which are the usual color SU(3)$_c$ and weak hypercharge U(1)$_Y$ groups.

It is of interest to explore ultraviolet completions of TC2 models in which one can show that the assumed symmetry breakings occur in the desired manner. We have carried out such a study (Ryttov and Shrock, Phys. Rev. D $\mathbf{82}$, 055012 (2010), arXiv:1006.5477).

The scale Λ_t is fixed in TC2 models by m_t, and the scale at which $\mathrm{SU}(3)_1 \otimes \mathrm{SU}(3)_2$ breaks to SU(3)$_c$ cannot be larger than this or else the $\mathrm{SU}(3)_1$ interaction would break before it could produce the desired $\langle \bar{t}t \rangle$ condensate. This yields an upper bound on the masses of the eight massive vector bosons in the coset $\mathrm{SU}(3)_1 \otimes \mathrm{SU}(3)_2/\mathrm{SU}(3)_c$ of order $\sim$ TeV.

TC2 models are constrained by ATLAS and CMS lower limits on axigluon (coloron) masses up to ~ 3 TeV. (Tevatron and LHC constraints discussed in talks by Martin, Mishra).

TC theories are also subject to constraints from precision electroweak data, in particular, the modification of the W and Z propagators, as described by the parameters $\Delta\rho = \alpha_{em} T$ and S (discussed in talk by Schaich). Exp. allowed oval region in (S,T) generally has $S \lesssim 0.2$ (depending on assumed SM m_H).

A naive perturbative estimate for technifermion contributions to S is

$$S_{TC,pert.} \simeq \frac{d_R\, N_D}{6\pi}$$

where d_R is the dimension of the technifermion rep. under G_{TC}, e.g., $d_R = N_{TC}$ for fundamental rep. However, as is well known, this perturbative estimate is not reliable, since TC is strongly coupled at scale m_Z.

If TC were QCD-like, nonperturbative effects would yield $S_{TC} \simeq 2S_{TC,pert.}$ (Peskin-Takeuchi, 1990), so clearly TC cannot be scaled-up QCD-like theory.

In general, the constraint from the S parameter remains a crucial and stringent one for TC/ETC theories. Interesting recent results on S from the Lattice Strong Dynamics Collaboration for quasi-conformal gauge theories (Schaich).

TC/ETC Model-Building with Color-Singlet Technifermions

Much TC/ETC model-building and phenomenology have been done with one-family TC (1FTC) (early study, Eichten, Hinchliffe, Lane, Quigg, RMP 56, 579 (1984)).

If one uses $G_{TC} = \mathrm{SU}(2)$, then the TC theory has $N_f = 2(N_c + 1) = 8$ technifermions, which is close to estimates of $N_{f,cr}$ and hence can plausibly produce the desired walking behavior.

However, $S_{TC,pert.} = 4/(3\pi) \simeq 0.4$, so one needs strong suppression of S.

1FTC models have a very large global chiral symmetry, with many pseudo-Nambu-Goldstone bosons (PNGBs) carrying color and charge. These models also predict technihadrons including color-octet techni-vector mesons. ATLAS and CMS data set limits on these for masses up to ~ 2 TeV.

One is thus motivated to revisit the one-doublet TC (1DTC) model, which uses a minimal technifermion content consisting of one (color-singlet) SU(2)$_L$ doublet with corresponding right-handed components:

$$F_L^\tau = \begin{pmatrix} F_1^\tau \\ F_2^\tau \end{pmatrix}_L \ \text{with hypercharge} \ Y_{F_L}$$

(τ = TC index) and

$$F_{1R}^\tau, \quad F_{2R}^\tau, \quad Y_{f_{iR}}, \quad i = 1, 2$$

Electric charge is vectorial $\Leftrightarrow$ $Y_{F_{1R}} = Y_{F_L} + 1$ and $Y_{F_{2R}} = Y_{F_L} - 1$.

If these two technifermions are in the fund. representation of G_{TC}, then the model would not have walking behavior. Approaches to getting walking: (i) include $N_{f,cr} - 2$ additional technifermions, taken to be SM-singlets; (ii) assign F_1, F_2 to higher-dimensional representations of G_{TC}, e.g., adjoint rep. of SU(2). Extensive studies of (ii) by Sannino and coworkers.

A recent study of a model that uses method (i) is Ryttov, RS, PRD 84, 056009 (2011), arXiv:1107.3572; further work in progress with Appelquist, Ryttov and Y. Bai.

Since certain ETC gauge bosons in a 1DTC model transform quarks into the (color-singlet) technifermions, these ETC gauge bosons are color triplets, and hence $[G_{ETC}, G_{SM}] \neq 0$.

In general, for this type of model, $G_{ETC} \supset \mathrm{SU}(3)_c \otimes G_{gen.} \otimes G_{TC}$, where $G_{gen.}$ is the gauged generational group.

If one took the simplest choice, $N_{TC} = 2$ and $Y_{F_L} = 0$, the TC theory would be free of anomalies in gauged currents. However, the most attractive channels would lead to the Majorana condensates ($\alpha,\ \beta$ are SU(2)$_L$ indices)

$$\langle \epsilon_{\alpha\beta}\epsilon_{\tau\tau'} F_L^{\alpha\tau\ T} C F_L^{\beta\tau'} \rangle, \quad \langle \epsilon_{\tau\tau'} F_{1R}^{\tau\ T} C F_{2R}^{\tau'} \rangle \ ,$$

Restricting to the fundamental rep., one may thus choose an SU(3) TC gauge group. Since this TC theory has an odd number of SU(2)$_L$ doublets, one must add another SU(2)$_L$ doublet to avoid a global Witten anomaly:

$$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L , \quad \psi_{1R} \ , \quad \psi_{2R}$$

This can be taken to be a singlet under both SU(3)$_c$ and SU(3)$_{TC}$. One can get $S_{TC,pert.} \lesssim 0.2$ here. Further details in paper. This type of model is interestingly different from one-family TC/ETC models and exhibits a number of intriguing features that merit further study.

Conclusions

- Technicolor continues to be an interesting and well-motivated possibility for dynamical electroweak symmetry breaking.

- Lattice simulations have made great progress in studying strongly coupled quasi-conformal gauge theories. The knowledge gained is valuable for the construction of walking TC theories. It is of interest to compare higher-loop continuum calculations of γ with lattice results.

- Studies of reasonably UV-complete TC/ETC theories have yielded insights into how to produce the observed fermion mass hierarchy (via sequential self-breaking of a strongly coupled chiral ETC gauge symmetry) and plausibly satisfy FCNC constraints (via residual approximate generational symmetries). However, predicting SM masses and mixing in detail is still a difficult challenge.

- Useful to pursue further TC/ETC model-building.

- Opportune time for this research, since data from the LHC should soon elucidate the source of electroweak symmetry breaking.